statstutor

community project

encouraging academics to share statistics support resources

Statistical Methods 10. Introduction to Analysis of Variance (ANOVA)

Based on materials provided by Coventry University and Loughborough University under a National HE STEM Programme Practice Transfer Adopters grant

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Reviewer: Ellen Marshall
University of Sheffield

Workshop outline

- Motivation for ANOVA
- Checking assumptions
- ANOVA using SPSS
- Multiple comparisons – post hoc tests

Participants should have previous experience of:

- Descriptive Statistics – see Workshop 3
- SPSS – see Workshop 7
- Two sample tests – see Workshop 8

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Example 1

- Amount of oil used by four machines (litres/week)
- Recorded over 6 sampled periods
- Does this sample data provide evidence that oil consumption differs between the machines?

$\Rightarrow$ Create summary statistics and error bar charts

$\Rightarrow$ Describe the data

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Oil data

Machine number gives 4 data groups (known as a **factor**)

Note: This example has the same number of data values for each group, but this is not necessary (as in the unpaired t-test)

Machine	**1**	**2**	**3**	**4**
Oil consumption	72	91	93	66
	64	78	75	55
	68	97	78	49
	77	82	71	64
	56	85	63	70
	95	77	76	68

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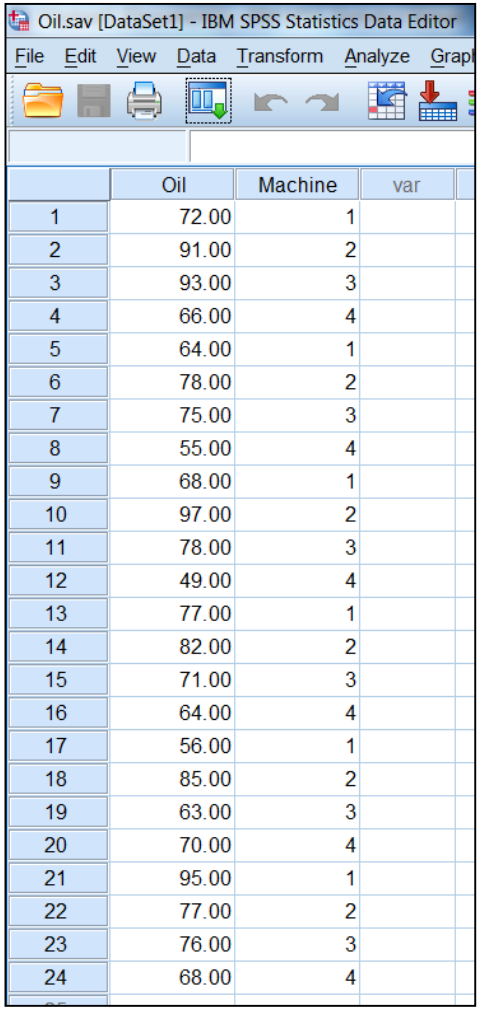
Oil data in SPSS

- Open the file Oil.sav
- Oil data is given in a single column with the *Machine* variable indicating the machine it refers to



	Oil	Machine	var
1	72.00	1	
2	91.00	2	
3	93.00	3	
4	66.00	4	
5	64.00	1	
6	78.00	2	
7	75.00	3	
8	55.00	4	
9	68.00	1	
10	97.00	2	
11	78.00	3	
12	49.00	4	
13	77.00	1	
14	82.00	2	
15	71.00	3	
16	64.00	4	
17	56.00	1	
18	85.00	2	
19	63.00	3	
20	70.00	4	
21	95.00	1	
22	77.00	2	
23	76.00	3	
24	68.00	4	

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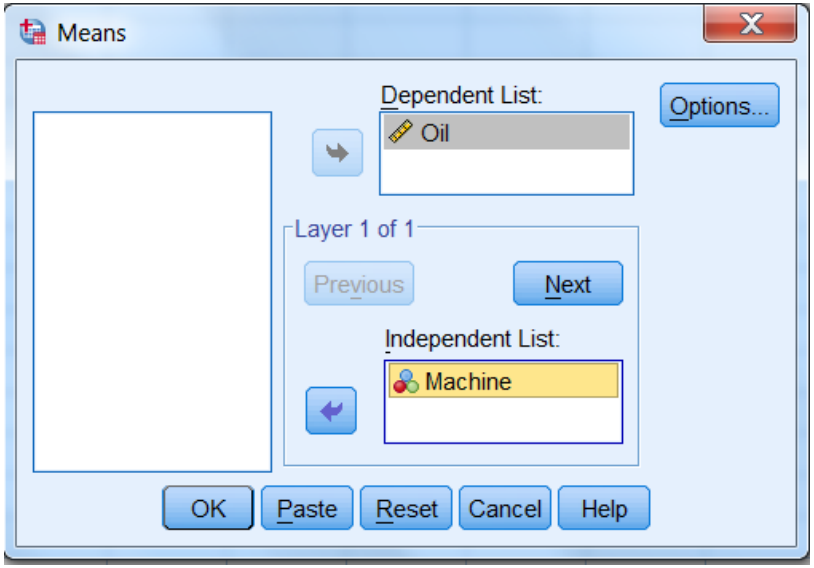
Simple statistics

- Analyze - Compare means – means
- Add *Oil* and *Machine* as shown



Report

Oil

Machine	Mean	N	Std. Deviation
1	72.0000	6	13.34166
2	85.0000	6	7.77174
3	76.0000	6	9.87927
4	62.0000	6	8.22192
Total	73.7500	24	12.60521

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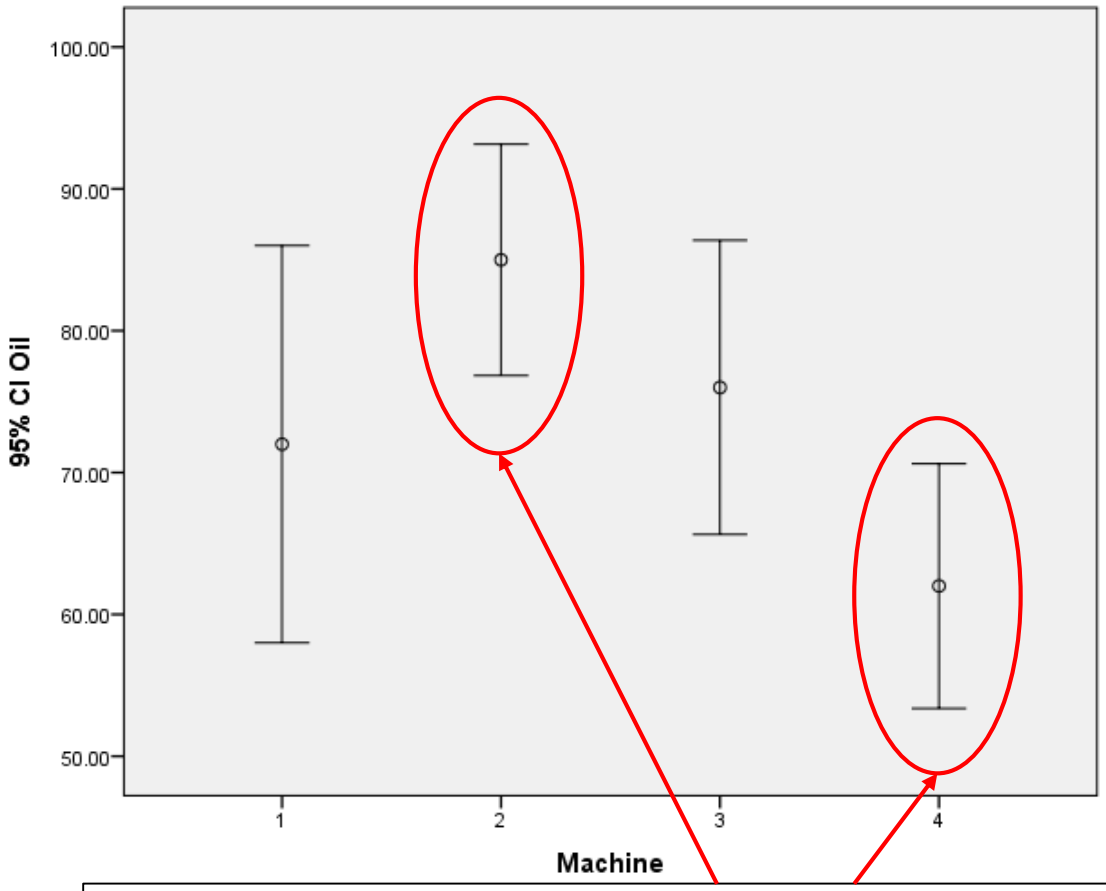
Error bar chart (Oil v. Machine)



Error bar charts are better for larger samples.

They show the means and their confidence intervals

Non-overlapping confidence intervals indicate possible significant differences

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Initial observations

- There appear to be differences between the sample means, i.e. variation between groups
- But there is also variation within groups
- Can we conclude that there are differences between groups (population means)?
- We need an objective approach – this is known as **ANOVA**

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Introduction to ANOVA

- ANOVA is a multiple group extension of the two sample independent t test used to compare two groups (population means)
- ANOVA is used to compare several groups (population means)
- Called ANOVA from **AN**alysis **O**f **VA**riance
- (The name is therefore a bit confusing because it appears to be a **means** test, not a variance test)

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Introduction to ANOVA

- Better than doing lots of two sample tests, e.g. 6 tests for 4 machines
- For every test, there is a probability that we reject H_0 when it is true
- This probability is 0.05 for testing at a significance level of 0.05
- Doing several tests increases the probability of making a wrong inference of significance (Type I error)
- E.g. for our example, the probability of a wrong inference, assuming they are all equally randomly distributed and that these events are independent is $1 - 0.95^6 = 1 - 0.735 = 0.265$, i.e. more than 1 in 4

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The ANOVA model

$$y_{ij} = \mu + m_i + e_{ij}$$

- y_{ij} denotes oil consumption for the j^{th} measurement of the i^{th} machine
- The parameter m_i denotes how the consumption for machine i differs from the overall mean μ
- e_{ij} denotes the error for the j^{th} measurement of the i^{th} machine
- The ANOVA model assumes that all these errors are normally distributed with zero mean and equal variances

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Testing

- In our example, we test the hypothesis:

$$H_0: m_1 = m_2 = m_3 = m_4 = 0$$

  Or, more simply, that the machine means are the same

- Intuitively, this is done by looking at the difference between means relative to the difference between observations, i.e. is the mean to mean variation greater than you would expect by chance?

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Assumptions

(Similar to the two-sample unpaired t-test)

1. The dependent values y_{ij} are normally distributed for each i. However, if there are many groups there is a danger of a Type I error.
2. The errors e_{ij} for the whole data set are normally distributed. But we must estimate the sample means ($\mu + m_i$) first. (This theoretically follows from Assumption 1, but it is worth testing separately with small samples.)
3. The variances of each group are equal

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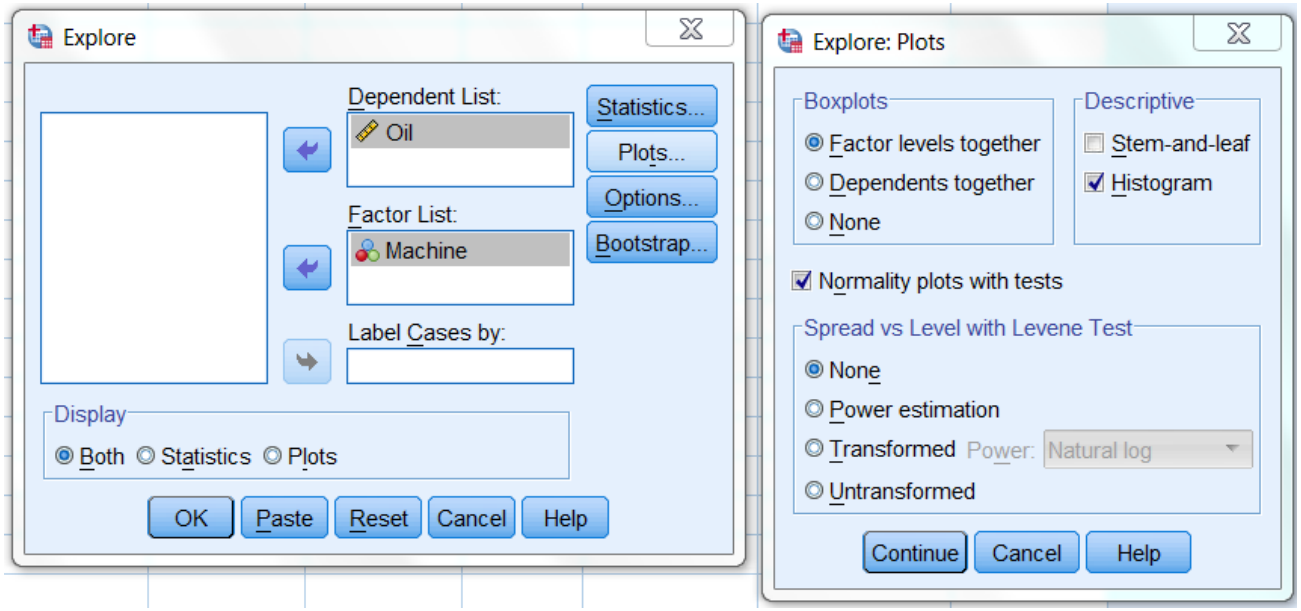
Assumption 1: Testing each group for normality

- Analyze – Descriptive Statistics – Explore
- Choose the variables as shown
- Select Plots… and choose Histogram and Normality plots with tests as shown



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Tests of Normality

	Machine	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
Oil	1	.187	6	.200*	.950	6	.741
	2	.167	6	.200*	.932	6	.593
	3	.253	6	.200*	.933	6	.607
	4	.263	6	.200*	.888	6	.310

a. Lilliefors Significance Correction
*. This is a lower bound of the true significance.

- Shapiro-Wilk test significance levels are all greater than 0.1 (look at this test first for small to medium sizes, up to one or two thousand)
- No evidence that individual machine data is not normally distributed

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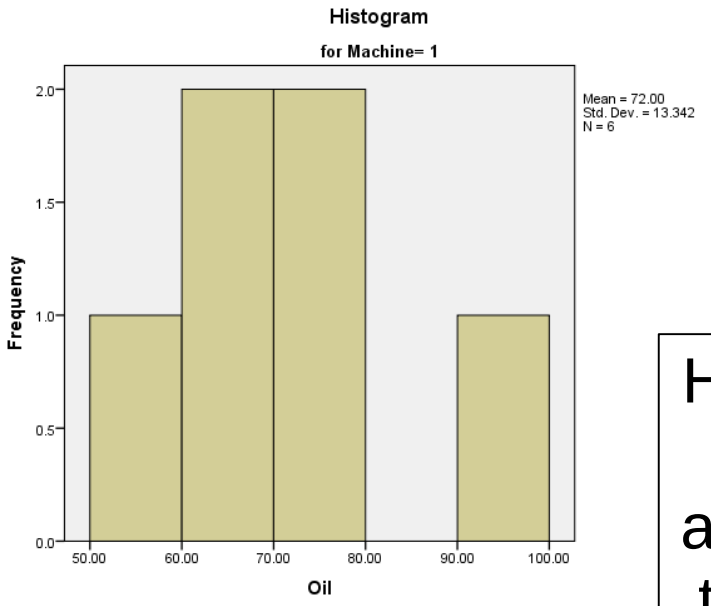

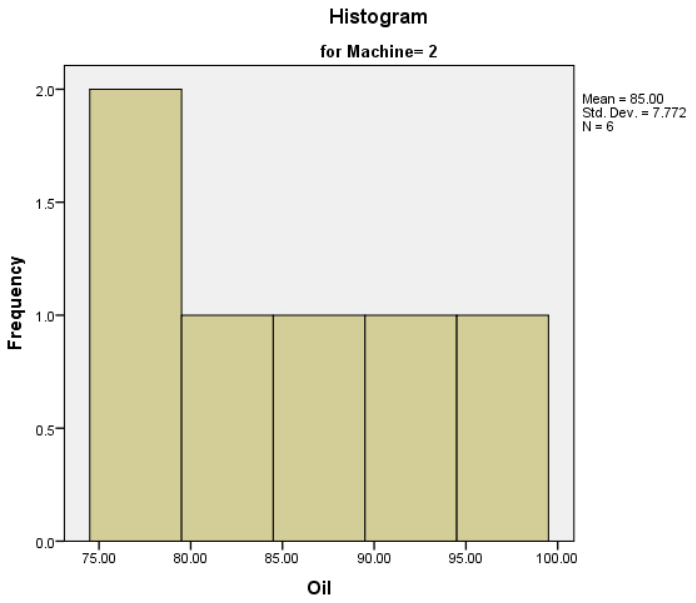

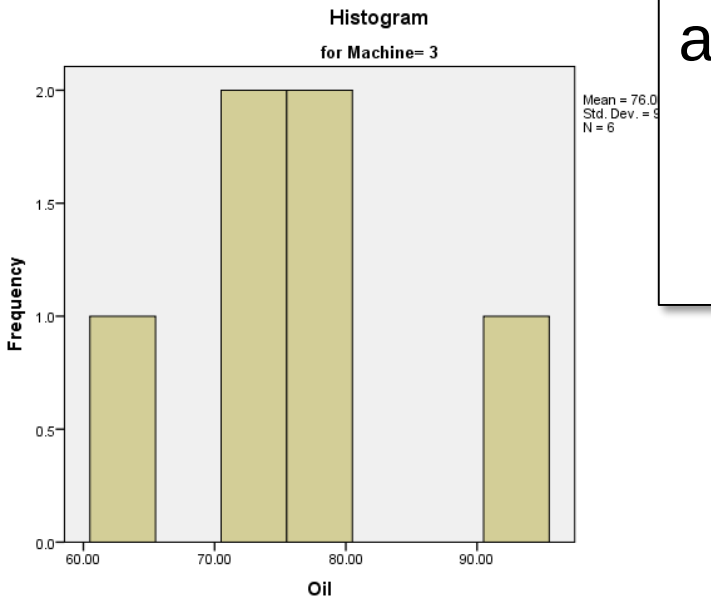

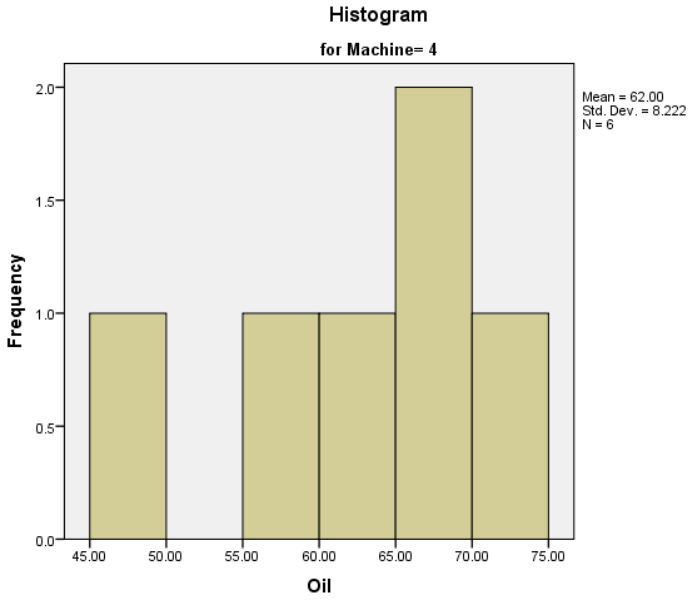

Histograms are acceptable, taking into account the small sample sizes

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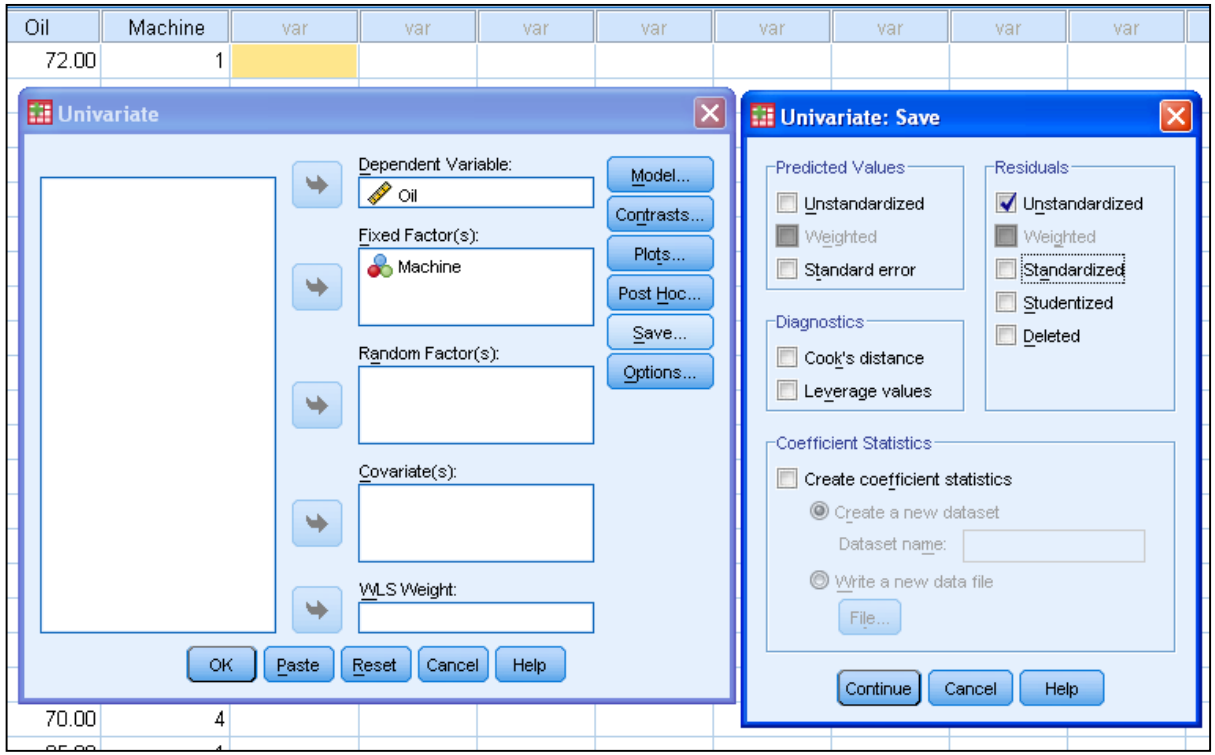
Assumption 2: Testing errors for normality

- First create the residuals
- Select Analyze – General linear model – Univariate
- Add the variables as shown
- Select Save…
- Choose Unstandard-ised residuals
- Based on estimates of m_i



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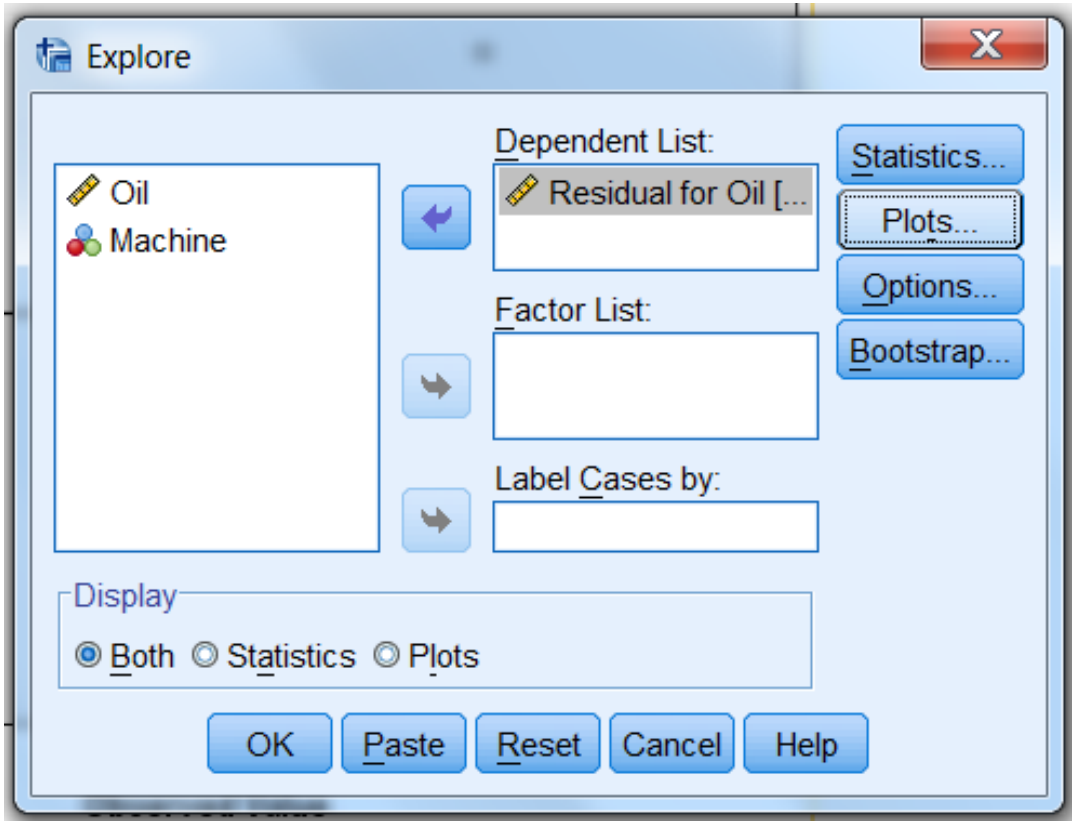
- Select Analyze – Descriptive Statistics – Explore
- Add the residual variable as shown
- Keep the Plots… settings as before



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Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Residual for Oil	.094	24	.200*	.972	24	.721

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

- Significance level of Shapiro-Wilk test is greater than 0.1
- No evidence that the residuals are not normally distributed
- However, a slightly higher threshold is required than usual because we have already estimated the group means $\mu + m_i$ (and thus reduced the degrees of freedom)

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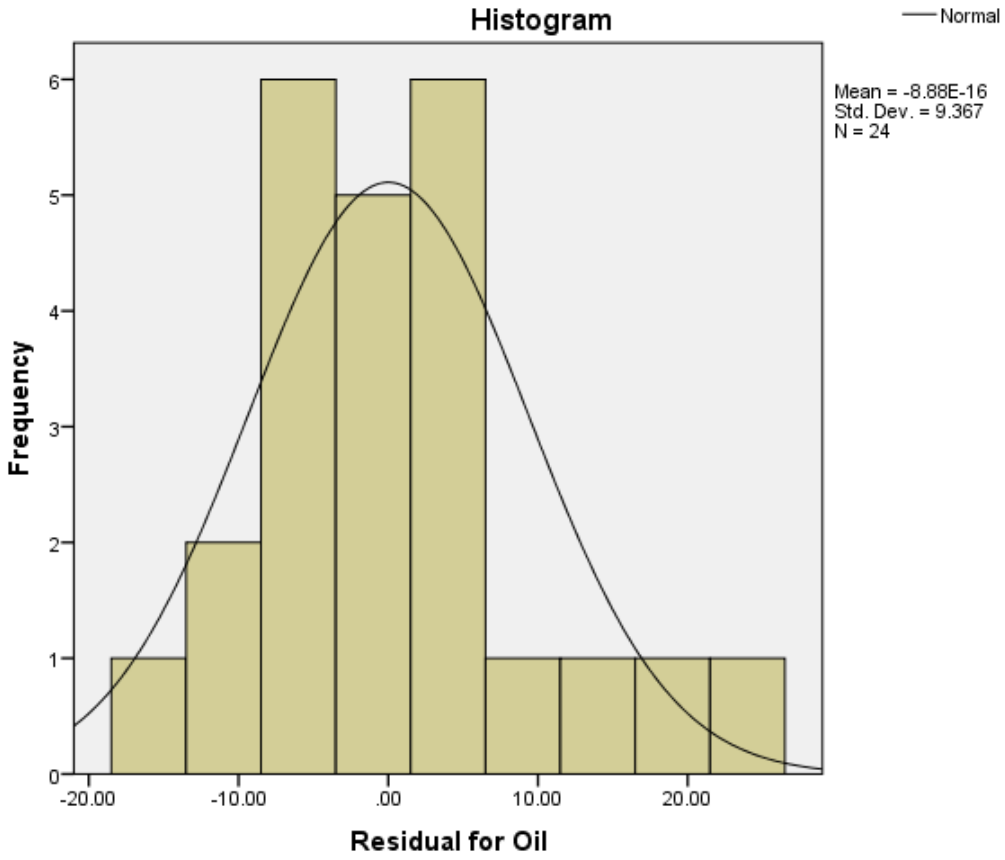
The histogram is again acceptable. The sample size is now 24. A normal curve approximation has been added using the Chart Editor window.



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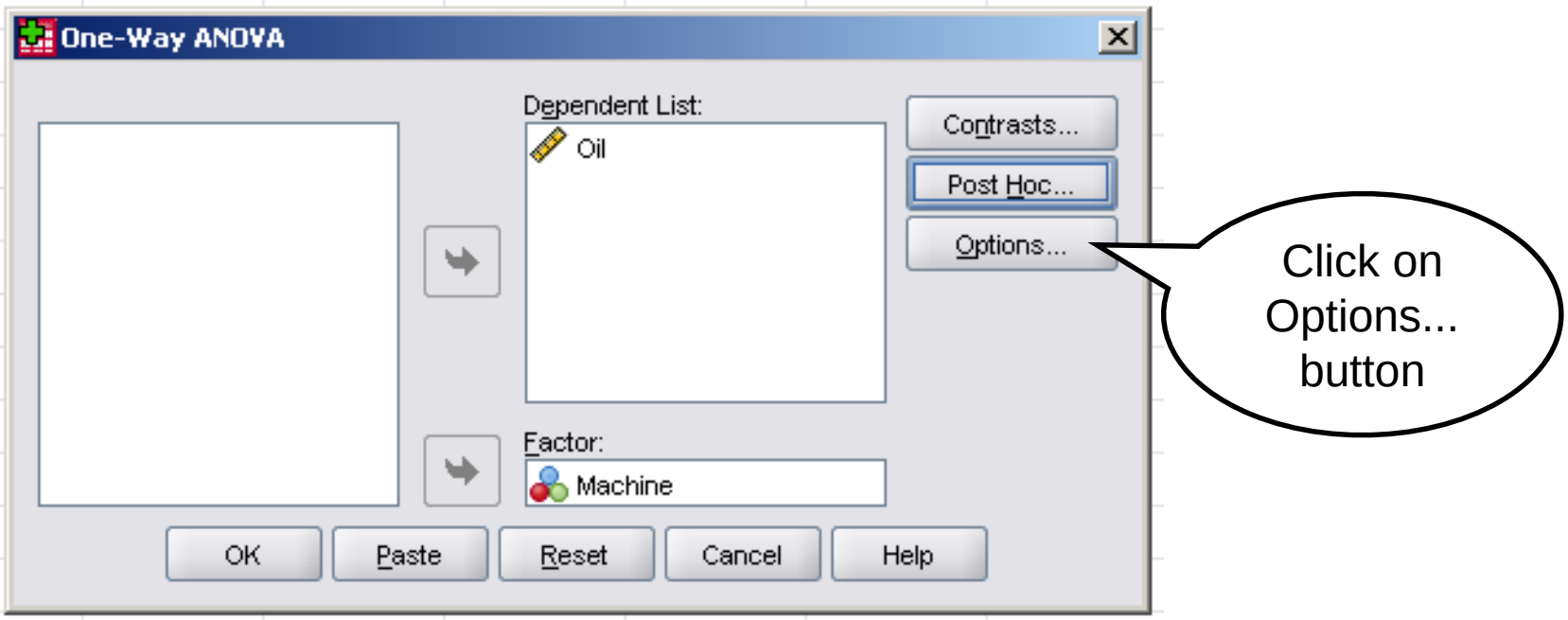
Assumption 3: Equal variances for Oil data

Analyze → Compare Means → One-Way ANOVA



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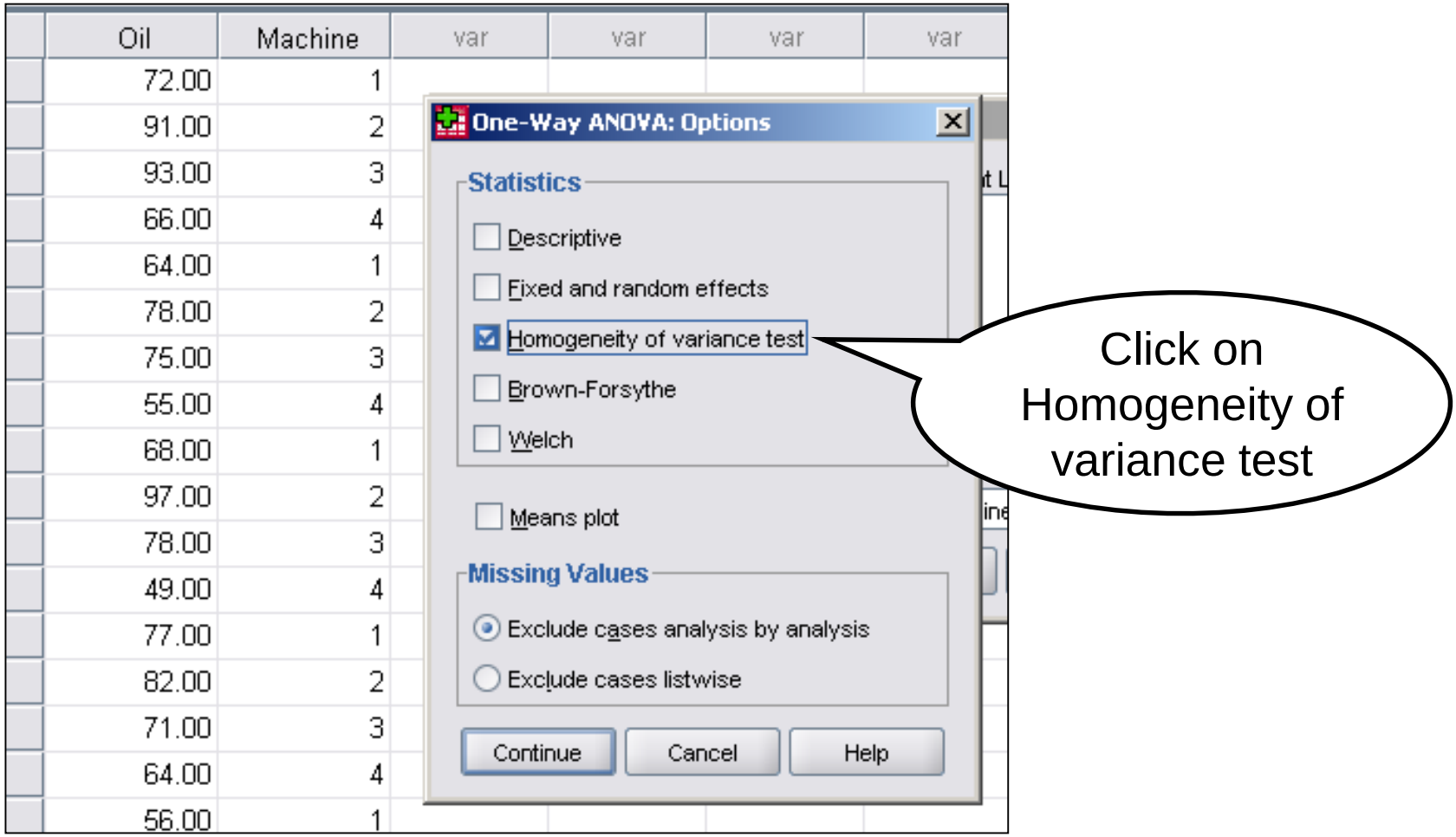

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- This carries out a Levene's test for homogeneity of variance
- Null hypothesis: the variances are equal

Test of Homogeneity of Variances

Oil

Levene Statistic	df1	df2	Sig.
.361	3	20	.782

- Significance value > 0.1 so we have no evidence to doubt assumption of equal variances

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Example 2

- A research project involving three different designs of a new product
- Tested by 60 people
- Each person was assigned to assess one product, providing in an overall performance score out of 100
- 20 people per product

$\Rightarrow$ Create summary statistics and an error bar chart

$\Rightarrow$ Describe the data

$\Rightarrow$ Test the ANOVA assumptions

$\Rightarrow$ Interpret the output

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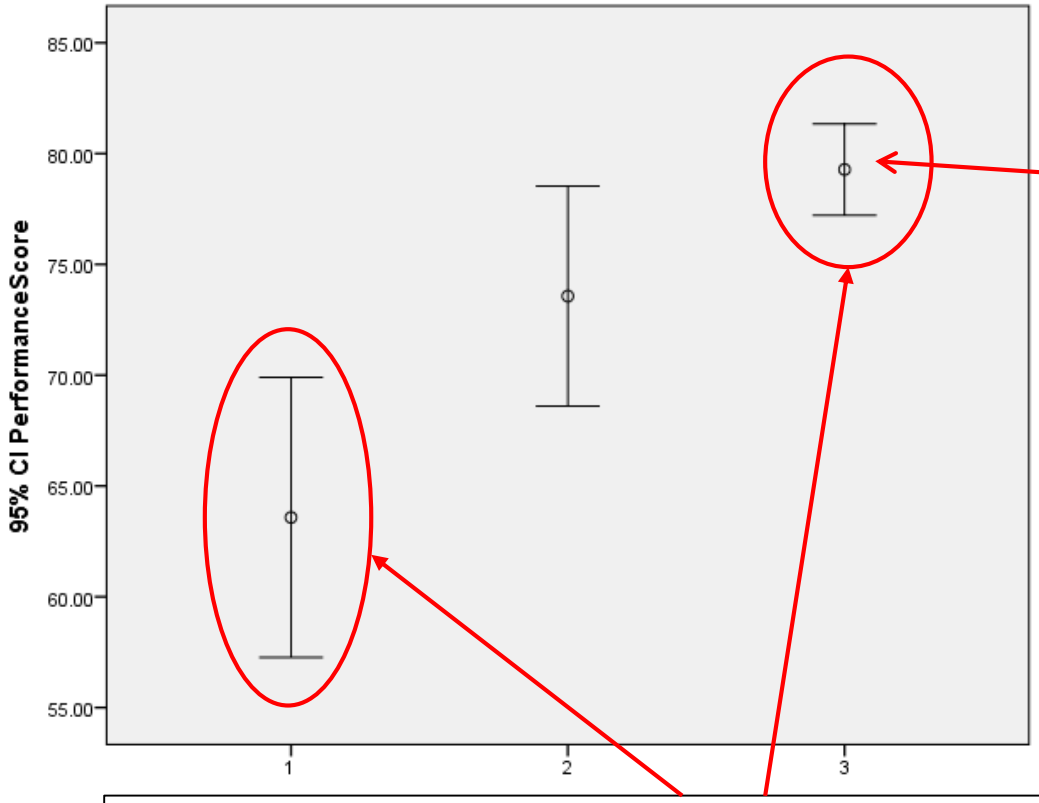
Error bar chart (*PerformanceScore* v. *Design*)



Performance scores for *Design 3* seems to be quite different from the other two groups, especially *Design 1*.

The variance of *Design 3* also seems to be smaller.

As before, these confidence intervals clearly don't overlap, indicating likely significant differences

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Check normality of each group

- Analyze – Descriptive Statistics – Explore
- Select *PerformanceScore* in the Dependent list and *Design* as the factor
- Select Normality plots with tests and Histograms under Plots…

Tests of Normality

		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Design	Statistic	df	Sig.	Statistic	df	Sig.
PerformanceScore	1	.139	20	.200*	.957	20	.494
	2	.134	20	.200*	.948	20	.344
	3	.153	20	.200*	.962	20	.582

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

- No evidence that individual groups are not normally distributed

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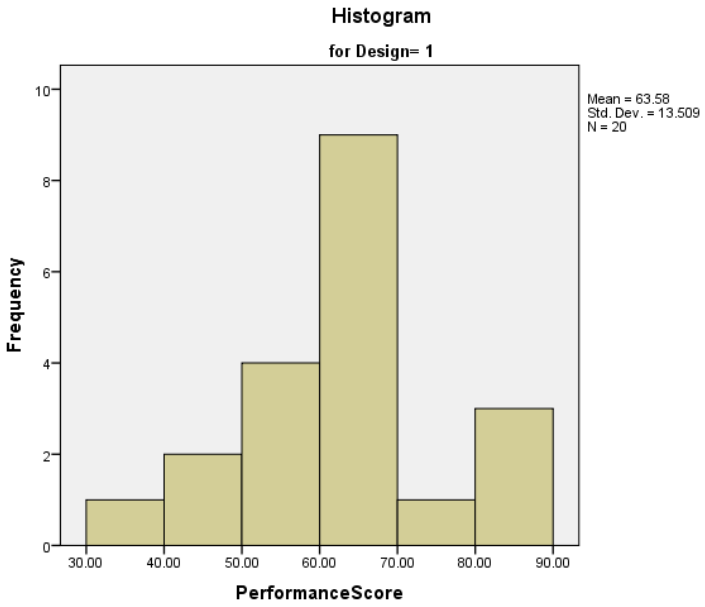
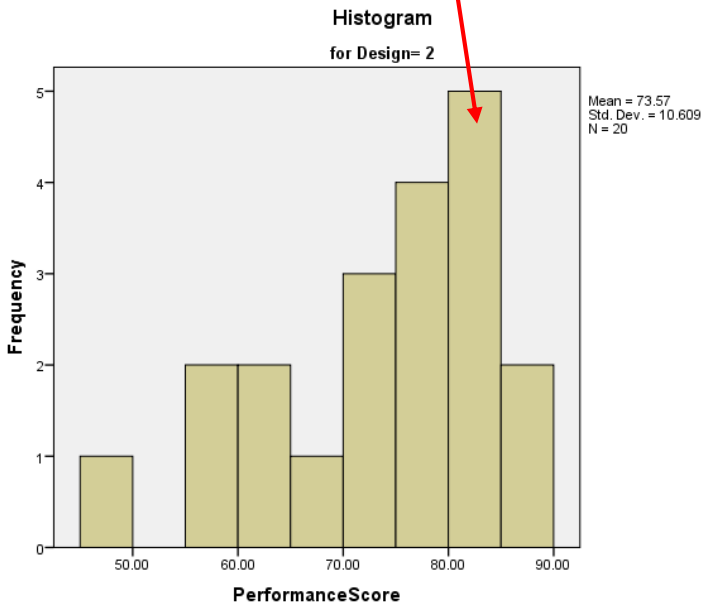
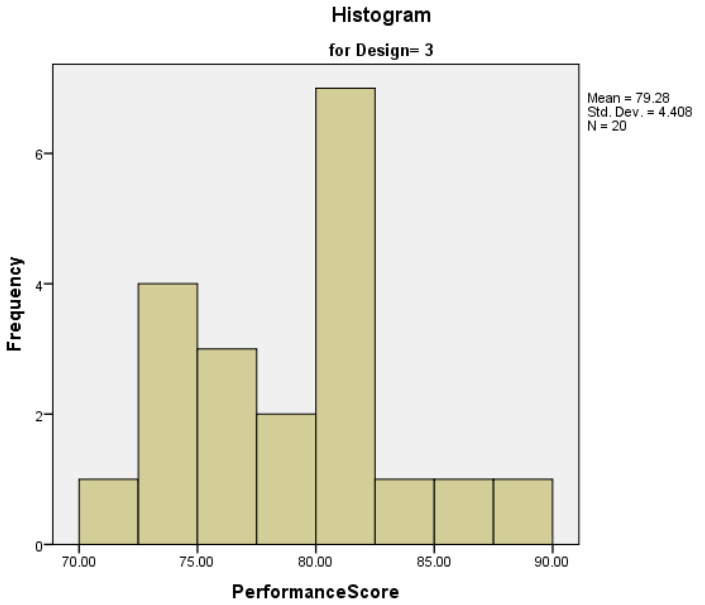
Histograms are fairly acceptable, although *Design 2* appears to have a slight negative skew (although it is less than twice its standard error)







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Normality of errors check

- Analyze – General Linear Model – Univariate
- Save… Unstandardised Residuals as before
- Analyze – Descriptive Statistics – Explore
- Select *Residual for PerformanceScore* as the variable
- Select Plots… Normality plots with tests

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Residual for PerformanceScore	.123	60	.025	.957	60	.032

a. Lilliefors Significance Correction

- Evidence that residuals are not normally distributed from Shapiro-Wilk test ($p < 0.05$)

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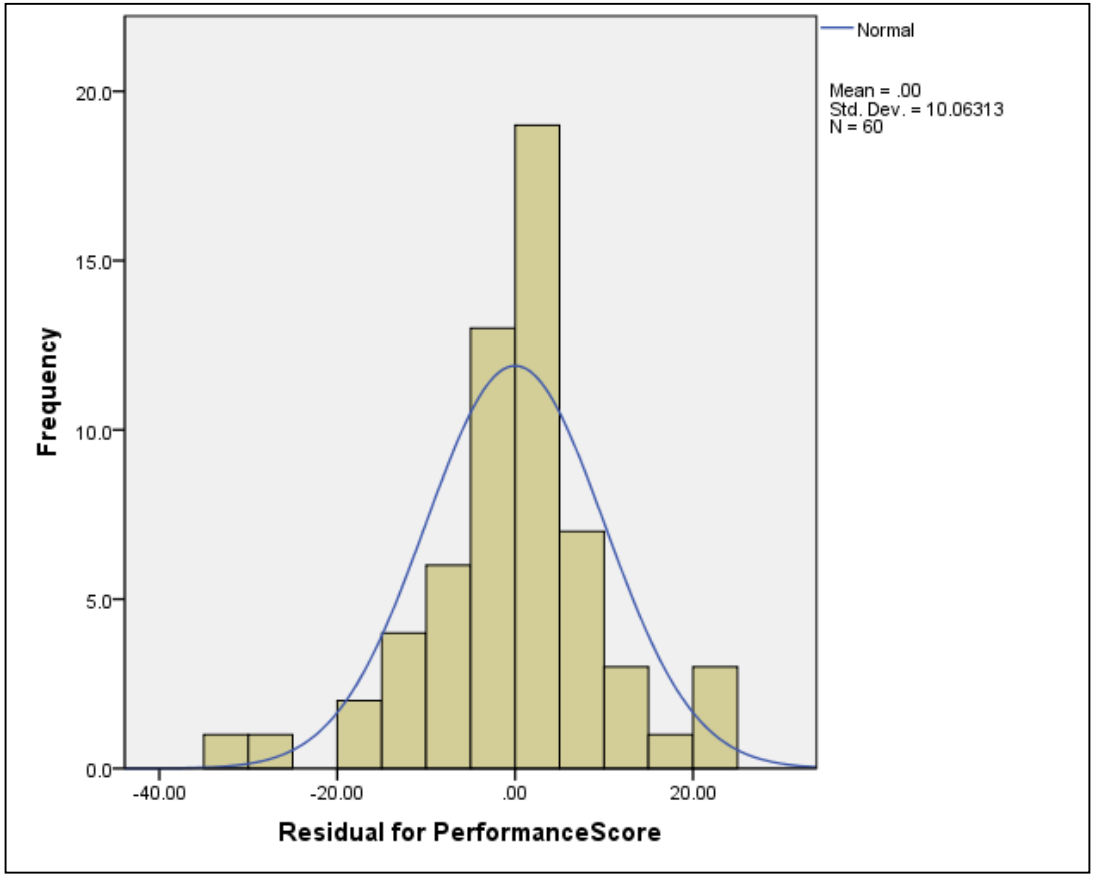
- Kurtosis looks a bit high – it is 1.553
- Its standard error is 0.608
- So it is more than twice its standard error



Equality of variances check

- Analyze – Compare Means – One-Way ANOVA
- Select Options… and Homogeneity of variance test

Test of Homogeneity of Variances

PerformanceScore

Levene Statistic	df1	df2	Sig.
4.637	2	57	.014

- Significance value < 0.05 so we do have evidence to reject the assumption of equal variances

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Robustness of ANOVA

- ANOVA is quite robust to changes in skewness but not to changes in kurtosis. Thus, it should not be used when:

$$\frac{|Kurtosis|}{Standard\ Error\ of\ Kurtosis} > 2$$

  for any group.

- Otherwise, provided the group sizes are equal and there are at least 20 degrees of freedom, ANOVA is quite robust to violations of its assumptions
- However, the variances must still be equal

Source:

Glass, G. V., Peckham, P. D. and Sanders, J. R. (1972) Consequences of failure to meet assumptions underlying the fixed effects analyses of variance and covariance, *Review of Educational Research*, 42(3), pp. 237-288.

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Robustness calculation for Example 2

Group	Kurtosis	Standard Error of Kurtosis	$\frac{\lvert Kurtosis \rvert}{Standard\ Error\ of\ Kurtosis}$
1	0.493	0.992	0.497 < 2
2	0.435	0.992	0.439 < 2
3	0.115	0.992	0.116 < 2

- Group sizes are equal
- Total degrees of freedom = 20 + 20 + 20 – 1 = 59 > 20
- All OK so far
- However, ANOVA cannot be used because the variances are not equal

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Summary of findings: ANOVA assumptions

Example	1	2
Normality of groups	No evidence of non-normality	No evidence of non-normality
Normality of residuals	No evidence of non-normality	Evidence of non-normality
Equality of variances	No evidence of non-equality	Evidence of non-equality
Robustness	N/A	Satisfied apart from non-equality of variances

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What if these assumptions are in doubt?

- If normality assumptions are in doubt:
  - Use a **non-parametric** test: Kruskal-Wallis (general) or Jonckheere-Terpstra (where the groups are in a sequence and you wish to look for a linear trend)
  - Select Analyze – Nonparametic Tests – Independent Samples… then select these tests on the Settings tabs after selecting Customise Tests
- If variances assumption in doubt:
  - Use the **Brown-Forsythe** or **Welch** test (the Welch test is more powerful except where there is an extreme mean with a large variance when the Brown-Forsyth is better)
  - Select ANOVA and click on Options… button and select the **Brown-Forsythe** and **Welch** options
  - Use the significance values there instead

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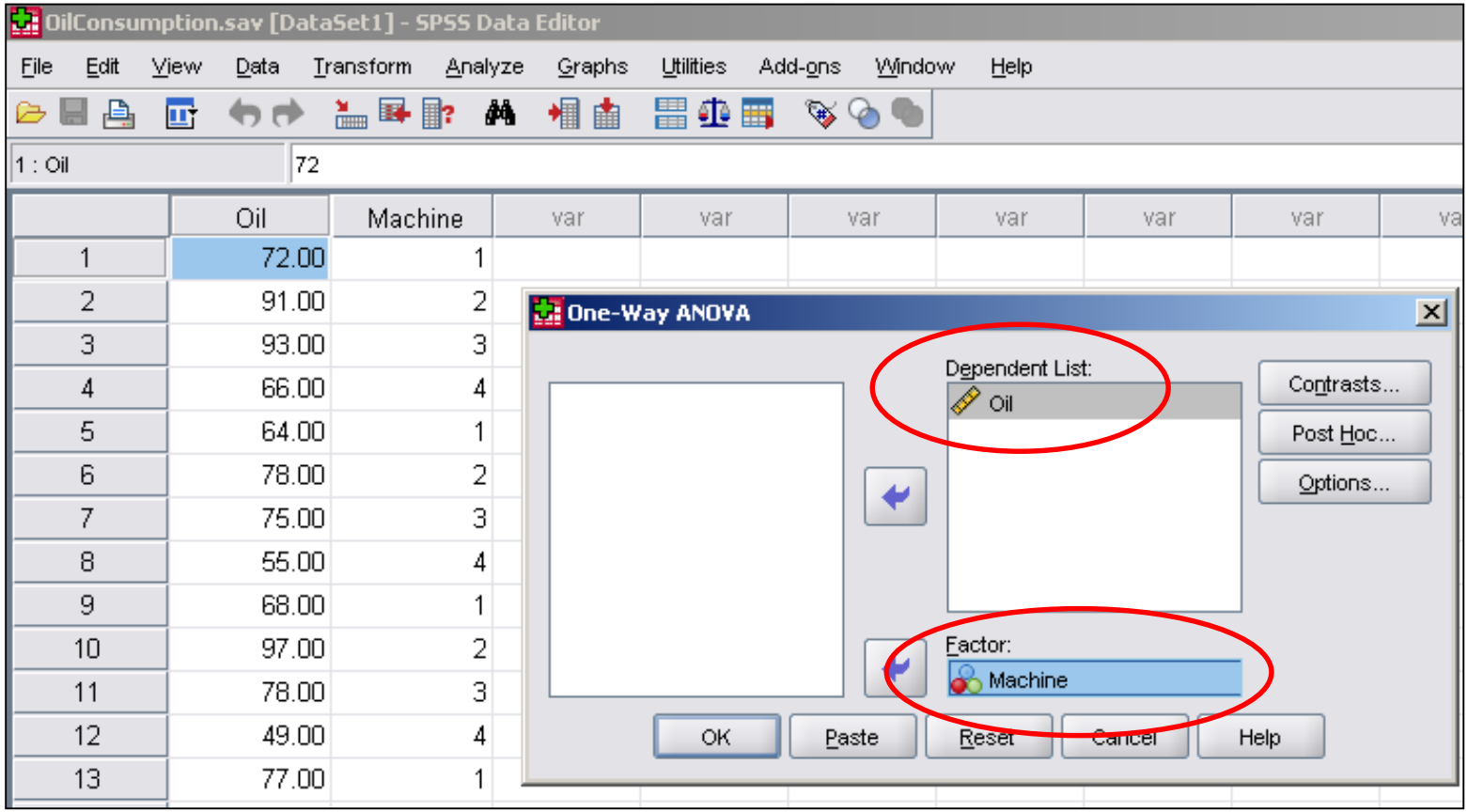
Example 1

- All 3 assumptions are OK so use normal ANOVA
- Analyze – Compare Means – One-Way ANOVA



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SPSS output

ANOVA

Oil

	Sum of Squares	df	Mean Square	F	Sig
Between Groups	1636.500	3	545.500	5.406	.007
Within Groups	2018.000	20	100.900		
Total	3654.500	23			

- Significant at 0.01
- So there is strong evidence of differences in mean oil consumption between the four machines

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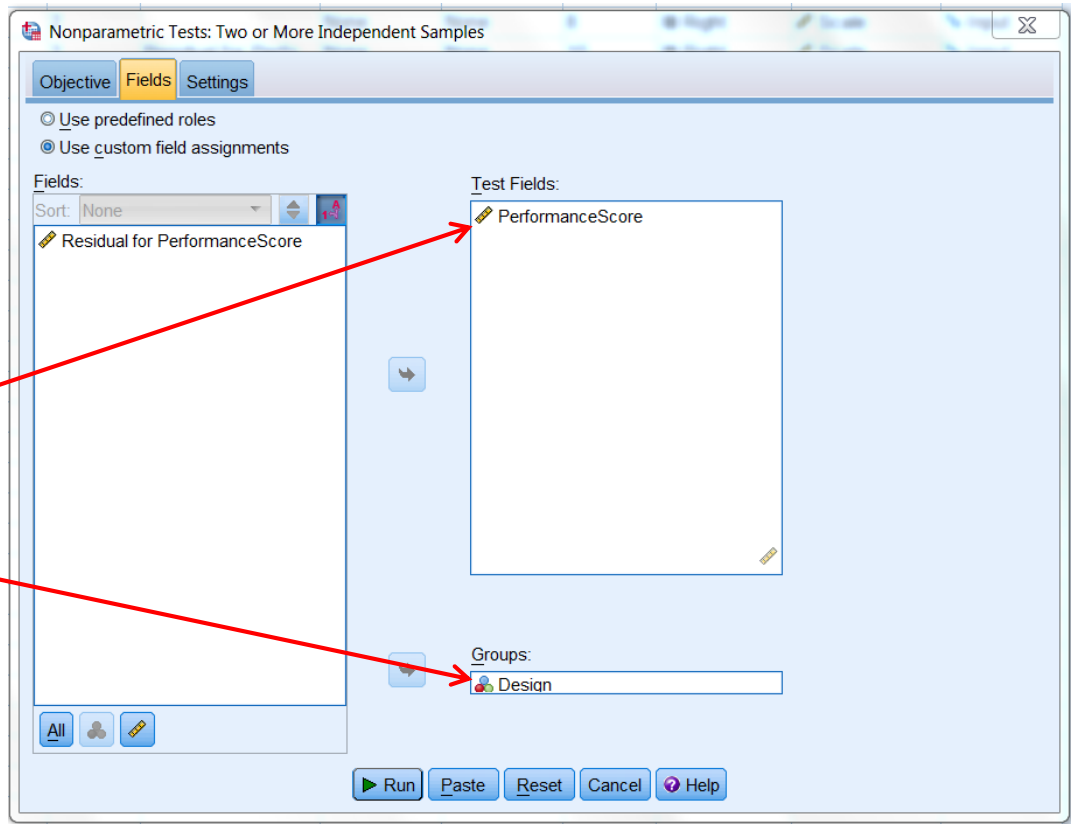
Example 2

- Normality cannot be assumed and groups are not ordered so use the Kruskal-Wallis test
- Select Analyze – Nonparametric tests – Independent Samples…
- Add *PerformanceScore* and *Design* on the Groups tab



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Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of PerformanceScore is the same across categories of Design.	Independent-Samples Kruskal-Wallis Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

- Give a p-value < 0.001
- Very strong evidence that there are differences between the groups

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However, ANOVA was robust for Example 2 apart from the differences in variances so we can also use the Brown-Forsythe or Welch test:

Robust Tests of Equality of Means

PerformanceScore

	Statistic[a]	df1	df2	Sig.
Welch	13.278	2	30.962	.000
Brown-Forsythe	12.048	2	40.540	.000

a. Asymptotically F distributed.

- Both tests are significant at the 0.001 level
- Thus there is very strong evidence that the means are not equal

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Multiple comparisons

- What if we conclude there are differences between the groups?
- We don't know where differences are!
- We can do **post-hoc** tests to compare each pair of groups
- Similar to 2-sample tests but adjusted for the multiple testing issue

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Which post hoc test?

- For equal group sizes and similar variances, use **Tukey (HSD)** or, for guaranteed control over Type I errors (more conservative), use **Bonferroni**
- For slightly different group sizes, use **Gabriel**
- For very different group sizes, use **Hochberg's GT2**
- For unequal variances, use **Games-Howell**

Source: (Field, 2013: 459)

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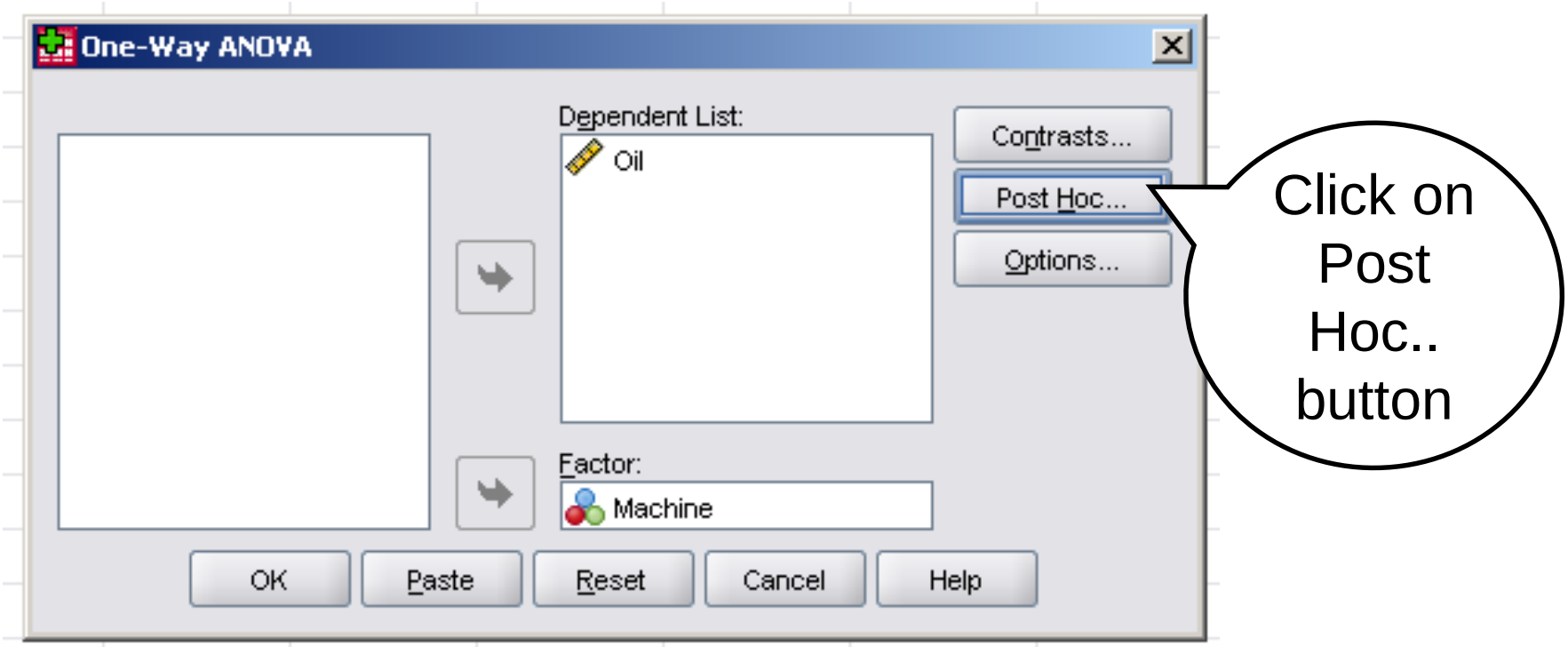
Example 1

Analyze – Compare Means – One-Way ANOVA



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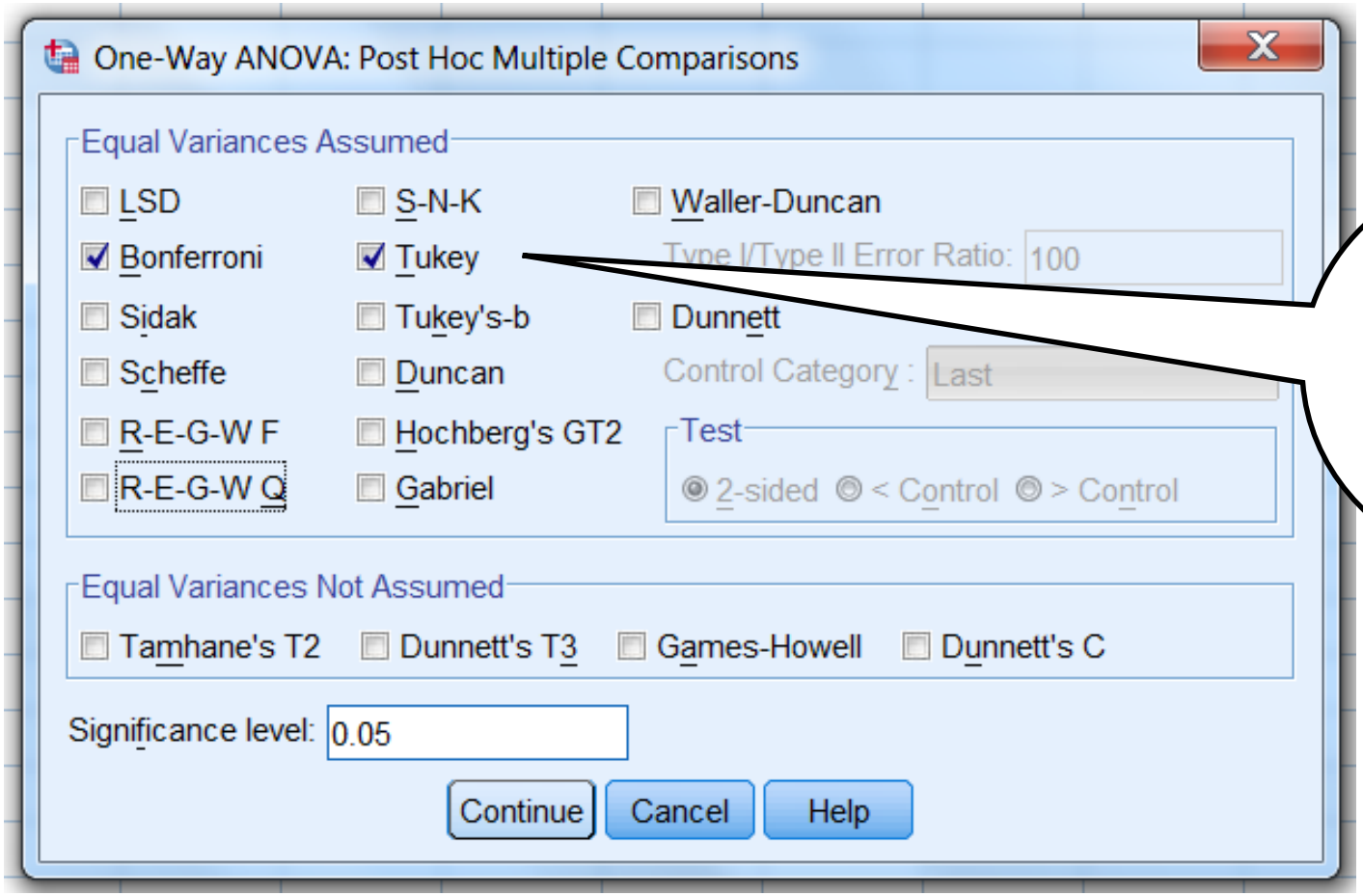
Multiple comparisons in SPSS



Choose Tukey and Bonferoni tests

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Multiple Comparisons

Dependent Variable:Oil

	(I) Machine	(J) Machine	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	1	2	-13.00000	5.79943	.146	-29.2322	3.2322
		3	-4.00000	5.79943	.900	-20.2322	12.2322
		4	10.00000	5.79943	.338	-6.2322	26.2322
	2	1	13.00000	5.79943	.146	-3.2322	29.2322
		3	9.00000	5.79943	.427	-7.2322	25.2322
		4	23.00000*	5.79943	.004	6.7678	39.2322
	3	1	4.00000	5.79943	.900	-12.2322	20.2322
		2	-9.00000	5.79943	.427	-25.2322	7.2322
		4	14.00000	5.79943	.107	-2.2322	30.2322
	4	1	-10.00000	5.79943	.338	-26.2322	6.2322
		2	-23.00000*	5.79943	.004	-39.2322	-6.7678
		3	-14.00000	5.79943	.107	-30.2322	2.2322
Bonferroni	1	2	-13.00000	5.79943	.219	-29.9756	3.9756
		3	-4.00000	5.79943	1.000	-20.9756	12.9756
		4	10.00000	5.79943	.600	-6.9756	26.9756
	2	1	13.00000	5.79943	.219	-3.9756	29.9756
		3	9.00000	5.79943	.818	-7.9756	25.9756
		4	23.00000*	5.79943	.005	6.0244	39.9756
	3	1	4.00000	5.79943	1.000	-12.9756	20.9756
		2	-9.00000	5.79943	.818	-25.9756	7.9756
		4	14.00000	5.79943	.153	-2.9756	30.9756
	4	1	-10.00000	5.79943	.600	-26.9756	6.9756
		2	-23.00000*	5.79943	.005	-39.9756	-6.0244
		3	-14.00000	5.79943	.153	-30.9756	2.9756

*. The mean difference is significant at the 0.05 level.

- Only significant difference for Tukey HSD is between Machines 2 and 4
- Strong evidence ($p < 0.01$) that Machine 2 uses more oil than Machine 4
- Significance levels are higher and confidence interval bounds are smaller than for Bonferroni, as expected

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Multiple comparisons conclusions

- Only significant difference is between Machines 2 and 4
- Strong evidence ($p < 0.01$) with both tests that Machine 2 uses more oil than Machine 4
- 95% confidence interval for difference between machines is approximately 7 to 39 litres/week
- No evidence of differences in oil usage between other machines (because all the other confidence intervals for Tukey HSD contain 0)

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Example 2

- As normality cannot be assumed, need to use nonparametric tests

Hypothesis Test Summary

	Null Hypothesis	Test	Sig.	Decision
1	The distribution of PerformanceScore is the same across categories of Design.	Independent-Samples Kruskal-Wallis Test	.000	Reject the null hypothesis.

Asymptotic significances are displayed. The significance level is .05.

Double-click on this note to open the Model Viewer dialogue box

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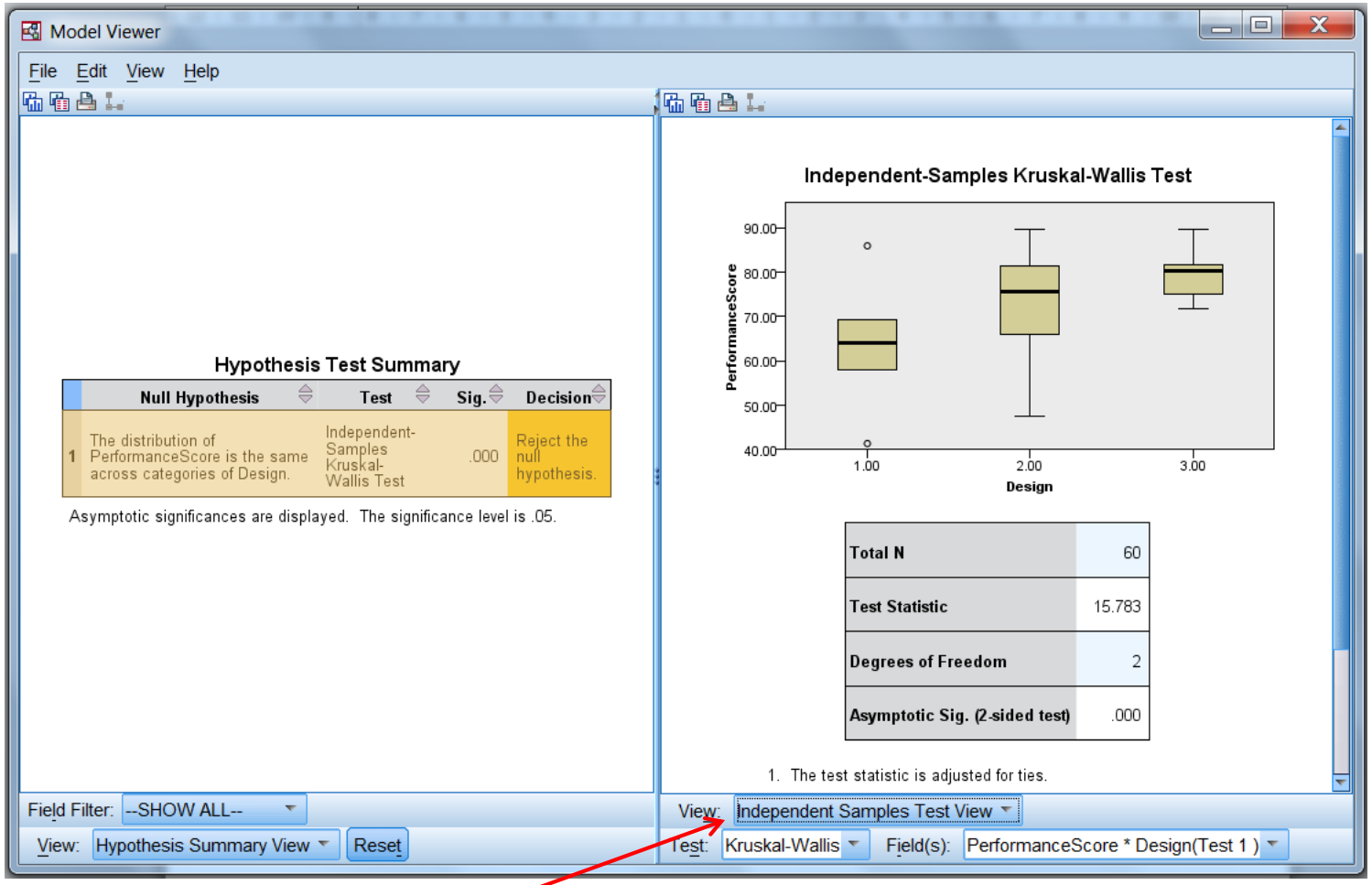
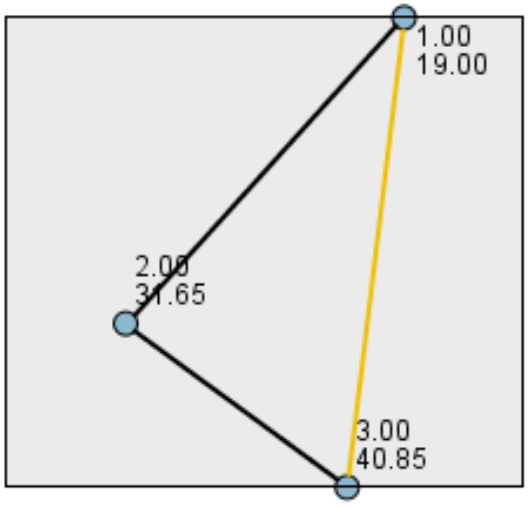


Change the view option to Pairwise Comparisons

- The adjusted significance values are corrected using an equivalent to the Bonferroni correction for parametric ANOVA
- Very strong evidence of a difference between groups 1 and 3
- Weak evidence of a difference between groups 1 and 2

Pairwise Comparisons of Design



Each node shows the sample average rank of Design.

Sample 1-Sam...	Test Statistic	Std. Error	Std. Test Statistic	Sig.	Adj.Sig.
0-1	-12.650	5.523	-2.291	.022	.066
0-2	-21.850	5.523	-3.956	.000	.000
1-2	-9.200	5.523	-1.666	.096	.287

Each row tests the null hypothesis that the Sample 1 and Sample 2 distributions are the same.
Asymptotic significances (2-sided tests) are displayed. The significance level is .05.

However, as ANOVA was robust apart from the equality of variances assumption we can also use the Games-Howell post hoc test:

More powerful conclusions than the nonparametric tests

Multiple Comparisons

PerformanceScore
Games-Howell

(I) Design	(J) Design	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
1	2	-9.98789*	3.84079	.035	-19.3762	-.5996
	3	-15.69947*	3.17733	.000	-23.6566	-7.7424
2	1	9.98789*	3.84079	.035	.5996	19.3762
	3	-5.71158	2.56883	.086	-12.1043	.6812
3	1	15.69947*	3.17733	.000	7.7424	23.6566
	2	5.71158	2.56883	.086	-.6812	12.1043

*. The mean difference is significant at the 0.05 level.

- Very strong evidence of differences between groups 1 and 3
- Evidence of differences between groups 1 and 2
- Weak evidence of differences between groups 2 and 3

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Recap

We have considered:

- Describing multiple groups:
  - Scatter plots
  - Means and standard deviations
  - Boxplots
- Checking assumptions:
  - Normality of each group (Shapiro-Wilk and Kolmogorov Smirnov)
  - Normality of errors (creating unstandardised residuals, then as above)
  - Equality of variances (Levene's test)
  - Robustness to violations of assumptions (kurtosis, group sizes and degrees of freedom)

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Recap (2)

- Carrying out the ANOVA test
- Unequal variances alternatives (Brown-Forsythe and Welch)
- Nonparametric alternatives: Kruskal-Wallis (general) and Jonckheere-Terpstra (linear)
- Post hoc tests (Tukey, Bonferroni, Gabriel and Hochberg's GT2)
- Unequal variances alternative (Games-Howell)
- Nonparametric alternatives (Kruskal-Wallis pairwise comparisons)

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